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Lecture 3: Basic Optics for Optical Sensing-I

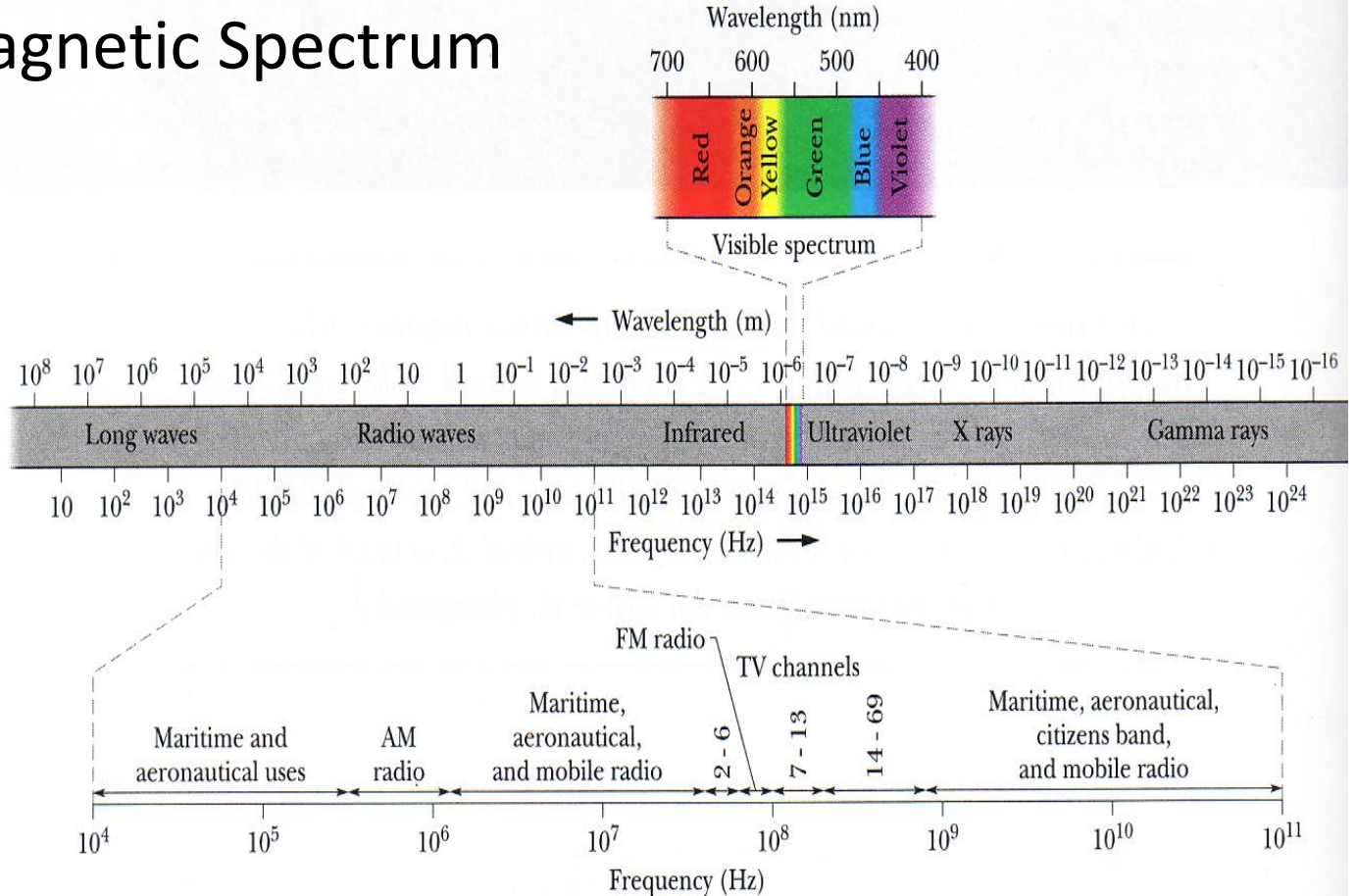
Electromagnetic Waves in Free Space & Poynting Vector

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The Electromagnetic Spectrum



Maxwell's Equations

$$\vec{\nabla} \cdot \vec{D} = \rho_{Ext} \quad \dots\dots\dots (i)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \dots\dots\dots (ii)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \dots\dots\dots (iii)$$

$$\vec{\nabla} \times \vec{H} = \vec{J}_{Ext} + \frac{\partial \vec{D}}{\partial t} \quad \dots\dots\dots (iv)$$

Constitutive Relations

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon \vec{E} \quad \dots\dots\dots (v)$$

$$\vec{B} = \mu_0 \vec{H} + \vec{M} \approx \mu \vec{H} \quad \dots\dots\dots (vi)$$

Where ρ_{Ext} & $\vec{J}_{Ext}$ $\equiv$ External charge and current densities

$\vec{E}$ = Electric Field, $\vec{H}$ = Magnetic Field, $\vec{D}$ = Displacement vector,

ϵ & μ = Permittivity & Permeability of dielectric medium

$\vec{B}$ = Magnetic induction, $\vec{P}$ = Polarization, $\vec{M}$ = Magnetization

Constants and Units

ϵ_0 , Permittivity of free space = $10^7 / 4\pi c^2 \text{ C}^2 / \text{Nm}^2$

μ_0 , Permeability of free space = $4\pi \times 10^{-7} \text{ N} / \text{A}^2$

c , Speed of Light in free space = $2.998 \times 10^8 \text{ m} / \text{s}$

$$\mu_0 \epsilon_0 = \frac{1}{c^2}$$

For charge and current free medium, $\rho_{\text{Ext}} \text{ \& } \vec{J}_{\text{Ext}} = 0$

Since, for most of the media, $\mu = \mu_0$ [\(Why is the effect of magnetic field negligible?\)](#)

Equations, (i), (ii), (iii)& (iv) can be modified as,

$$\vec{\nabla} \cdot \vec{E} = 0 \quad \dots\dots\dots \text{(ia)} \qquad \vec{\nabla} \cdot \vec{B} = 0 \quad \dots\dots\dots \text{(iia)}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \dots\dots\dots \text{(iiia)} \qquad \vec{\nabla} \times \vec{B} = \mu_0 \epsilon \frac{\partial \vec{E}}{\partial t} \quad \dots\dots\dots \text{(iva)}$$



Take $\vec{\nabla} \times$ of equation (iiia):
$$\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = \vec{\nabla} \times \left[-\frac{\partial \vec{B}}{\partial t} \right]$$

Change the order of differentiation on the RHS:

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\frac{\partial}{\partial t} [\vec{\nabla} \times \vec{B}]$$

But, from Eq. (iva):
$$\vec{\nabla} \times \vec{B} = \mu\epsilon \frac{\partial \vec{E}}{\partial t}$$

Therefore,
$$\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\frac{\partial}{\partial t} \left[\mu\epsilon \frac{\partial \vec{E}}{\partial t} \right]$$

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\mu_0 \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

Using the identity $\vec{a} \times [\vec{b} \times \vec{c}] = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$ (Very useful, Prove it !!)

We get $\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] \equiv \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E}$

Hence, $\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$

But, from Eq. (ia), $\vec{\nabla} \cdot \vec{E} = 0$

Therefore, $\nabla^2 \vec{E} = \mu_0 \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$ or $\nabla^2 \vec{E} = \frac{1}{v^2} \frac{\partial^2 \vec{E}}{\partial t^2}$ **Wave Equation**

Similarly,

$$\nabla^2 \vec{H} = \frac{1}{v^2} \frac{\partial^2 \vec{H}}{\partial t^2}$$

(Prove it!!)

The Solution: Plane Waves

For free space, $v=c$

- Solutions must be plane waves

$$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)}$$

$$\mathbf{B} = \mathbf{B}_0 e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)}$$

$$\omega = ck$$

- $\mathbf{E}_0$ and $\mathbf{B}_0$ are not completely free
- Must satisfy all of Maxwell's equations

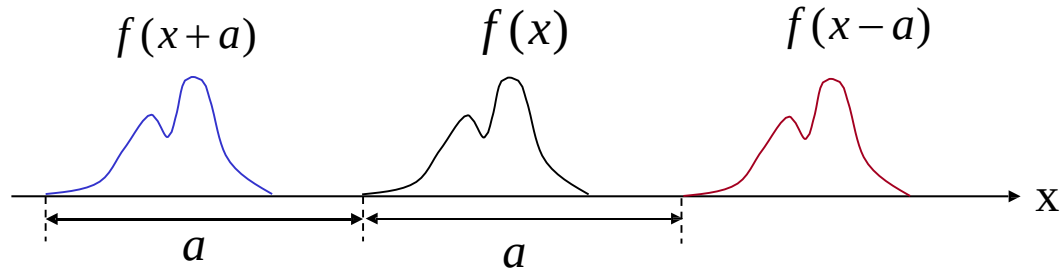
$$\nabla \cdot \mathbf{E} = 0 \Rightarrow \mathbf{k} \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0 \Rightarrow \mathbf{k} \cdot \mathbf{B} = 0$$

$\mathbf{E}$ and $\mathbf{B}$ are
perpendicular to $\mathbf{k}$

Why $(kx - \omega t)$?

Answer: to describe wave traveling in the positive x direction?



Upon shifting x by a the function origin moves to $x-a$!

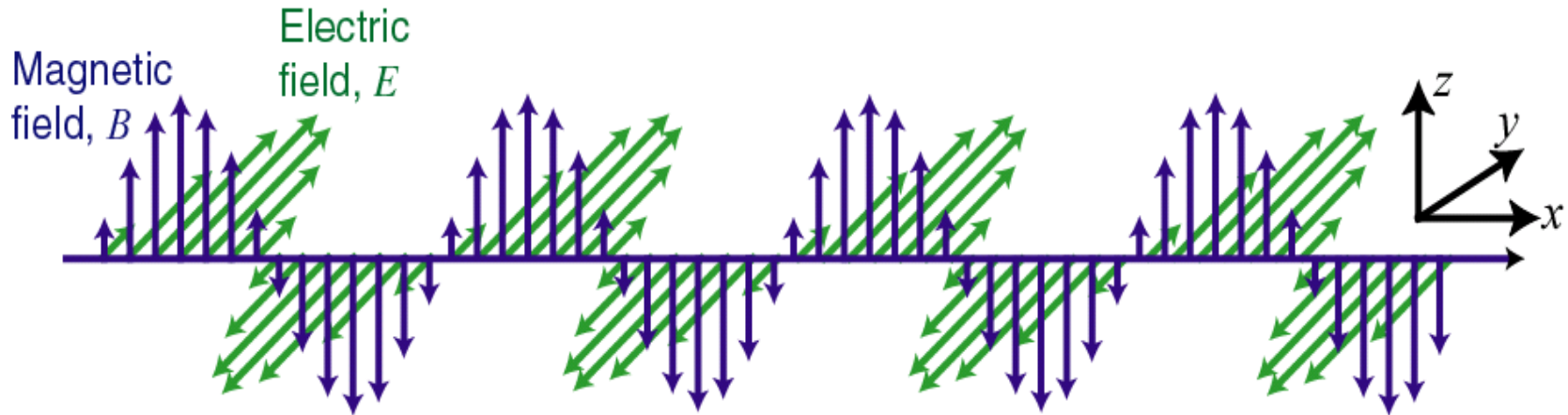
Take $a=vt$ and get the wave propagation in the positive x direction !

In our case: $f(x) = \exp(ik(x - ct)) = \exp(i(kx - \omega t))$

- $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \rightarrow \mathbf{k} \times \mathbf{E} = \omega \mathbf{B}$

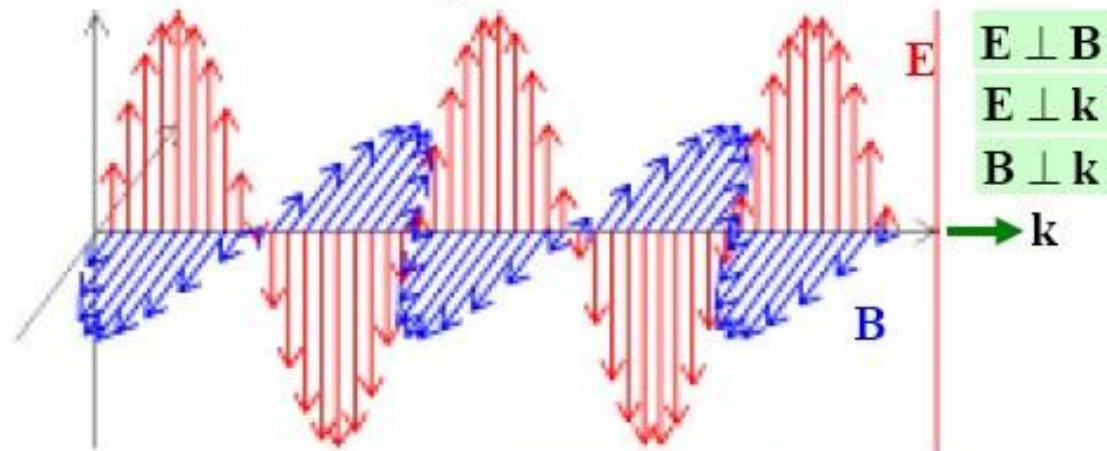
- $\nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \rightarrow \mathbf{k} \times \mathbf{B} = -\frac{\omega}{c^2} \mathbf{E}$

$\mathbf{E}$ and $\mathbf{B}$ are perpendicular to each other



The electric and magnetic fields are **in phase**.

- EM waves in free space is **transverse**



- From $\mathbf{k} \times \mathbf{E} = \omega \mathbf{B}$ and $\omega = ck \Rightarrow |\mathbf{E}| = c|\mathbf{B}|$

- If you want $\mathbf{H}$,

$$|\mathbf{H}| = \frac{|\mathbf{B}|}{\mu_0} = \frac{|\mathbf{E}|}{c\mu_0} = \frac{|\mathbf{E}|}{Z_0}$$

Vacuum
impedance
(377Ω)

$|\mathbf{E}| = |\mathbf{B}|$ if we
were using CGS

Why is the effect of magnetic field negligible?

The force on a charge, q , is:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

$\vec{F}_{\text{electrical}}$ $\vec{F}_{\text{magnetic}}$

where $\vec{v}$ is the charge velocity

Taking the ratio of the magnitudes of the two forces:

$$\frac{F_{\text{magnetic}}}{F_{\text{electrical}}} \leq \frac{qvB}{qE}$$

$$\begin{aligned} |\vec{v} \times \vec{B}| &= vB \sin \theta \\ &\leq vB \end{aligned}$$

Since $B = E/c$:

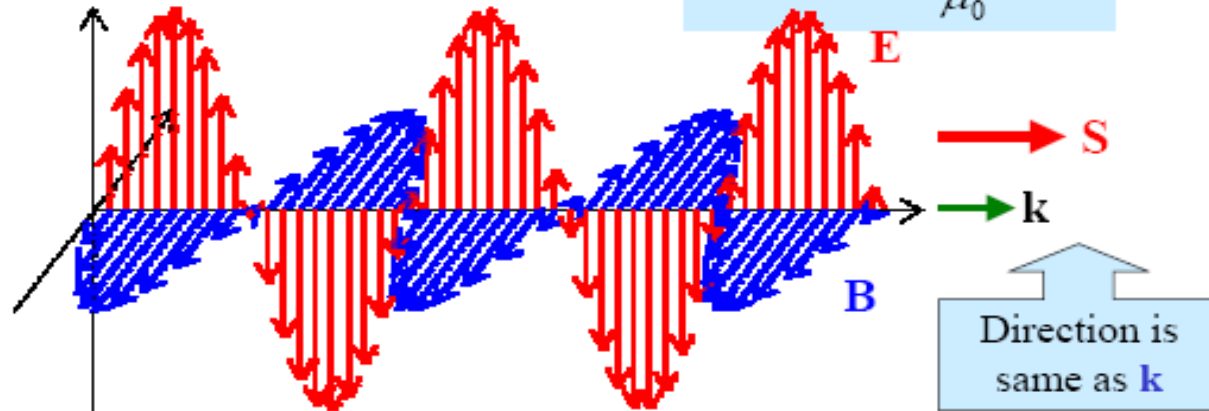
$$\frac{F_{\text{magnetic}}}{F_{\text{electrical}}} \leq \frac{v}{c}$$

So, as long as a charge's velocity is much less than the speed of light, we can neglect the light's magnetic force compared to its electric force.



Energy Flow: Poynting Vector

- Poynting vector is defined by $\mathbf{S} = \mathbf{E} \times \mathbf{H} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$



- Using $|\mathbf{H}| = \frac{|\mathbf{E}|}{Z_0} \Rightarrow |\mathbf{S}| = |\mathbf{E}||\mathbf{H}| = \frac{E^2}{Z_0}$

- Units: E in V/m, Z_0 in ohms $\rightarrow S$ in W/m^2

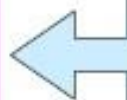
■ Poynting vector tells us:

- Energy is flowing in the direction of $\mathbf{k}$
- Energy flow density is $|\mathbf{S}| = E^2/Z_0$
- If we average over time,

$$\langle |\mathbf{S}| \rangle = \left\langle \frac{E_0^2 \cos^2(\mathbf{k} \cdot \mathbf{x} - \omega t)}{Z_0} \right\rangle = \frac{E_0^2}{2Z_0}$$

$$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)}$$

Average energy flow
density of EM waves
in free space



Summary

1. Starting from the Maxwell's Equations we deduced the wave equation and showed that it is transverse in nature.
2. The Energy of the wave flows in the direction of the Poynting Vector.

Thank You